Pattern Optimization of Intentional Blade Mistuning for the Reduction of the Forced Response Using Genetic Algorithm

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This paper investigates how intentional mistuning of bladed disks reduces their sensitivity to unintentional random mistuning. The class of intentionally mistuned disks considered here is limited, for cost reasons, to arrangements of two types of blades (A and B, say). A two-step procedure is then described to optimize the arrangement of these blades around the disk to reduce the effects of unintentional random mistuning. First, a pure optimization effort is undertaken to obtain the pattern(s) of the A and B blades that yields small/the smallest value of the largest amplitude of response to a given excitation in the absence of unintentional random mistuning using Genetic Algorithm. Then, in the second step, a qualitative/quantitative estimate of the sensitivity for the optimized intentionally mistuned bladed disks with respect to unintentional random mistuning is performed by analyzing their amplification factor, probability density function and passband/stopband structures. Examples of application with simple bladed disk models demonstrate the significant benefits of using this class of intentionally mistuned disks.

Key Words: Intentional Mistuning, Unintentional Random Mistuning, Monte Carlo Simulation, Probability Density Function, Passband/Stopband Structure

Nomenclature

A : Mistuned blade having natural frequencies 5% lower than tuned one

$A_{\max}^{(i)}$: Maximum amplitude of blade response on disk i

B : Mistuned blade having natural frequencies 5% higher than tuned one

C : Tuned blade

c : Damping coefficient of blade (N·s/m)

F_0 : Magnitude of exciting force (N)

g : Generation number of genetic algorithm

$g_{\max}$: Maximum number of generation

k_c : Coupling stiffness in blade-to-blade (N/m)

k_t : Stiffness of blade (N/m)

m_t : Blade mass (kg)

N : Total number of blades

n_{pop} : Population number of genetic algorithm

p_c : Crossover probability

p_m : Mutation probability

$p_s(i)$: Selection probability

r : Exciting engine order

σ : Standard deviation of unintentional random mistuning

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1. Introduction

In a dynamic analysis of a turbomachinery rotor, one traditionally has assumed that the blades are identical. The assumption of cyclic symmetry enables analysts to reduce the computational time considerably by modeling a single sector rather than modeling the entire blade assembly. However, in practice there are small differences in the structural and/or geometrical properties of individual blades, which are referred to as blade mistuning. Much of the vast literature on this topic (Kasa and Kielb, 1984, 1985 ; Wei and Pierre, 1988a, 1988b, 1990 ; Mignolet at al., 1998a, 1998b) has assumed these differences to be small and to arise either during the manufacturing process or as a result of in-service wear. These variations destroy the cyclic symmetry of the system and may change qualitatively its dynamic behavior. In particular, mistuning may lead to a confinement of the vibration energy to a few blades or even a single blade. As a result, mistuning localizes the response of the bladed disks to a few blades, creating a "vibration concentration" in the like of a stress concentration due to defects.

The motivation for considering such small variations is that their effects on the forced response of bladed disks can be extremely large as often reported in the above studies. Moreover, it has been recognized that the major source of high cycle fatigue is blade mistuning. Interestingly, the large sensitivity of the tuned system to these small variations has been linked (Wei and Pierre, 1988a) to its high level of symmetry.

In this light, it would appear beneficial to design bladed disks not to be tuned, i.e., to exhibit intentional mistuning, to reduce the sensitivity of their forced response to unintentional random mistuning, which can occur unavoidably in the manufacturing process and/or in service wear variations. Certainly, the consideration of intentional mistuning is not new : Kaza and Kielb (1984, 1985) demonstrated the value of alternate mistuning to raise the flutter speed. However, in the context of forced response, it is only recently that Castanier and Pierre (1997, 1998) and Kenyon and Griffin (2000) have investigated the use of harmonic patterns of mistuning and have shown that the magnification of the forced response due to unintentional random mistuning can be significantly reduced. (Castanier and Pierre, 1997, 1998) In particular, it was observed in these studies that the magnitude of the intentional mistuning must typically be of an order of or larger than that of the unintentional one for this design strategy to yield a benefit. Other studies of amplitude magnification (Ottarsson and Pierre, 1995 ; Kruse and Pierre, 1996 ; Mehmed and Murthy, 1988) have also demonstrated that some mistuning causes an increase in peak response while further mistuning often reverses the effect and decreases the sensitivity.

Note then that the use of different sets of blades on the same rotor is a complex and/or expensive proposition. Namely, for disks with inserted blades, it implies the manufacturing of different types of blades, the tracking of these different variants and the implementation of a systems which assures that the right blades have been inserted in the right slots. But, for blisks (bladed disk), the manufacturing issues may be less severe, although there are definite costs associated with the programming of different types of blade geometries on the manufacturing machines, and possibly more importantly with the necessary inspections for certification. In light of the above comments, it is highly desirable to optimize the benefit-over-cost ratio of intentional mistuning.

In this paper, the pattern optimization of intentional mistuning for bladed disks to reduce their sensitivity of unintentional random mistuning by an optimization strategy is the focus of the present investigation. The simple model (single and two-degree-of-freedom per blade) of bladed disks have been considered in this paper. There are two-step procedures in optimization strategy. First, a pure optimization effort is undertaken by Genetic Algorithm to obtain the pattern(s) that yields small/the smallest value of the largest amplitude of forced response to a given excitation in the absence of unintentional random mistuning. Then, in the second step, the sensitivity and robustness of the optimal pattern (i.e., intentionally mistuned bladed disks) with respect to unintentional random mistuning is identified by comparing the amplification factor as a function of unintentional random mistuning level, probability density function and passband/stopband structures of optimal pattern with those of other patterns and tuned blade.

2. Optimization Approach

In view of the complexity and cost of intentional mistuning, one should optimize the pattern to reduce as much as possible the amplification of the forced response to a given excitation. Accordingly, it is suggested here that an acceptable intentional mistuning pattern, i.e., maximizing the benefit-over-cost ratio, must be such that (a) it yields a large decrease in sensitivity to unintentional random mistuning, and (b) it involves a minimum number of types of blades, ideally 2.

It is proposed here to address this optimization problem sequentially. That is, the disk will first be assumed to support only two different types of blades (blades A and B, say) and their arrangement that yields the smallest amplification of the forced response due to unintentional random mistuning will be sought. Having established the increased cost of this mistuned design over a tuned one (blades C, say), the process can then be repeated with three types of blades, then four and so on until the cost increase is no longer justified by the reduction of sensitivity to mistuning.

The process described above has some rather dramatic computational implications. Indeed, for each mistuning pattern considered, it requires the determination of the largest amplitude of blade response that can be observed on a disk exhibiting both intentional and unintentional random mistuning. At this point in time, however, reliable estimates of this largest amplitude can only be obtained by time consuming Monte Carlo simulations. Monte Carlo simulations can only be performed after a problem has been detected and on very simple dynamic models of the bladed disk. Accordingly, a straightforward application of the proposed optimization strategy could only be done for very simple bladed disk models. It is thus proposed to proceed slightly differently by (1) performing the optimization in the absence of unintentional random mistuning, and (2) obtaining a qualitative/quantitative estimate of the sensitivity of a given intentionally mistuned disk to additional unintentional random mistuning.

In this manner, the optimization effort as step (1) requires only one forced response evaluation per intentionally mistuned disk considered, as opposed to an entire population.

Even with this simplification, care must be taken in selecting the optimization algorithm, as mistuning has a very nonlinear effect on the forced response. By switching the order of the blades around the disk, dramatic differences can be obtained in the variability of the blade-to-blade amplitudes of vibration as exemplified in particular by the harmonic mistuning analysis of Mignolet et al. (1998a). It might thus be suspected that there exists a series of local optima in the complex, high dimensional space over which the optimization must take place. In this light, the Genetic algorithm (GA) (Golderberg, 1989 ; Gen and Cheng, 1997) was used as optimization algorithm in this paper. Specifically, GA was used to find the pattern of two types of blades (blades A and B, say) that yields the largest reduction in the maximum forced amplitude in the absence of unintentional random mistuning.

3. Standard Genetic Algorithm

Genetic optimization algorithms are based on the transformation of a population of individual objects/design (here, bladed disks) through a series of generations using the Darwinian principle of reproduction and naturally occurring genetic operations of selection, crossover, and mutation. GA are particularly well suited for the present effort because the design variables only admit discrete values (i.e. a specific blade is only of type A or B) and there are multiple optima in pattern optimization of bladed disk (see References (Golderberg, 1989 ; Gen and Cheng, 1997) for further details).

The standard genetic algorithm (SGA) used here relies on a population of n_{pop} bladed disks (referred to as "chromosomes"), each of which is a random arrangement of N "genes"(the type A or B of the different blades). Accordingly, each bladed disk can be characterized by a sequence of N A and B letters (i.e., AABBBBAABABBA⋯), which evolves from one generation to the next according to the rules of selection, crossover, and mutation until all chromosomes yield essentially similar values of the fitness or objective function (selected here as the maximum amplitude of blade forced response in a frequency sweep at the selected engine order).

In the fitness proportionate selection used here, each individual bladed disk i in the population is first assigned a probability of selection $p_s(i)$ given as Eq. (1).

$$p_s(i)=1-\frac{A_{\max}^{(i)}}{\sum_{j=1}^{n_{pop}} A_{\max}^{(j)}} \tag{1}$$

where $A_{\max}^{(i)}$ denotes the maximum amplitude of blade forced response on disk i. Then, n_{pop}, individuals of the population at generation g are selected, according to the probabilities $p_s(i)$, for further potential genetic processing (i.e., crossover and mutation), to generate the population $g+1$. Note that population members with a higher fitness are likely to be selected more often than those with a lower fitness.

Crossover is the general process by which two chromosomes (the "parents") are split into segments and recombined to form two new chromosomes (the "offspring"). The single point crossover technique is used in the present investigation. Note that this process does not necessarily take place for all pairs of chromosomes forming the population: rather the crossover only occurs for each pair with a fixed probability p_c.

Mutation is the process by which the genes of the offspring are varied from their parent counterparts. In the present binary situation, wherein blades are either of type A or B, the mutation was accomplished by independently allowing, with probability p_m, each of the genes to switch type. This probability plays a very important role in the convergence of the genetic algorithm to an optimum, or near optimum solution. Indeed, if p_m is large, the mutation process is very frequent and the diversity of the population is high. This situation is desirable in the first few generations to span properly the space of possible solutions but, toward the end of the process, a high mutation rate may force the disappearance of the optimum solution. Accordingly, the probability of mutation was selected here as an exponentially decreasing function of the generation number g as Eq. (2).

$$p_m(g)=p_m(0)\exp[-\tau g/g_{\max}] \tag{2}$$

Table 1 Values of the parameters of the standard genetic algorithm

Population size (n_{pop})	35
Chromosome length (N) =Number of blades on disk	12
Crossover probability (p_c)	0.70
Mutation probability (see Eq. (2))	$p_m(0)=0.99$ $\tau=5$
Max. number of generations ($g_{\max}$)	500

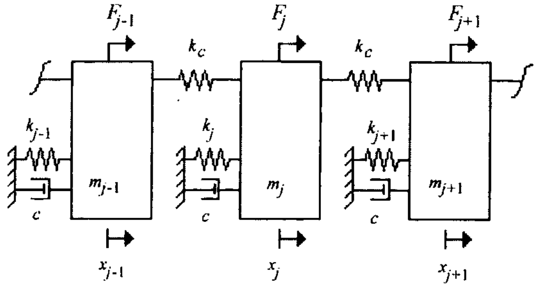
where $g_{\max}$ is the maximum number of generation.

Experimentation with the above algorithm has shown that the chromosome with the highest fitness (disk with the minimum amplitude of blade forced response) may disappear from one generation to the next due to the operations of crossover and mutation. To remedy this situation, the one elite reservation strategy was used according to which the best disk is retained unchanged from one generation to the next. It means that it is automatically selected and no mutation or crossover is performed on it.

Shown in Table 1 are the values of the GA parameters that led to the optimum results presented in this paper. These values were obtained after several trials and were retained because they led to the smallest value of the largest amplitude of blade response.

4. Application to Simple Bladed Disk Model

The simple structural model shown as Fig. 1 (one-degree-of-freedom per blade model) was first considered with two values of the coupling stiffness (large and weakly couple) to gain some perspective on the feasibility and benefits of the proposed optimization effort. Also, the two-degree-of-freedom per blade model (Mignolet at el., 1999) was considered in this paper. Although simple, this one-degree-of-freedom model has been found (Griffin and Sinha, 1985; Sinha, 1986; Wei and Pierre, 1988a, 1988b; Mignolet and Lin, 1997; Lin and Mignolet, 1996) to provide a good qualitative description of the damaging effects of mistuning in bladed disk. Specifically, each of the N blades is represented as a



$$F_j = F_o \cos\left(\omega t + \frac{2\pi r(j-1)}{N}\right)$$

$m_t = 0.0114$ kg, $k_t = 430{,}300$ N/m, $c = 0.143$ N·s/m,
$N = 12$ blades, $F_o = 1$ N, $r = 4$,
k_j : normal distribution of mean k_t and standard deviations

Fig. 1 Single-degree-of-freedom per blade disk model

single mass (m) which is connected to the ground (i.e. the disk) and the aerodynamic and structural coupling between blades are modeled by springs (k_c) and dashpots (c_c). In the sequel, it is assumed that the coefficient vanishes in this paper. The values of mass (m=0.0114 kg), damping (c=0.143 Ns/m, approximately 0.1% of the critical value) and stiffness (k_t=430,300 N/m) have already been used in a previous investigations to model a high-pressure turbine stage used in this paper. The computations proceeded as follows. The bladed disk model and excitation engine order were first selected. The SGA described above was then used to obtain the intentionally mistuned bladed disk pattern formed of blades A and B such that the maximum of its response over the entire frequency range was the smallest possible. This optimization was accomplished with the parameter values given in Table 1.

To assess the convergence of the SGA, the minimum, maximum, and mean value of fitness (largest amplitude of blade response) in the population was monitored as a function of the generation number. A typical plot of the evolution of these three values is shown in Fig. 2. Note that the minimum value is associated with the optimum A/B pattern. Thus, it is clearly seen from Fig. 2 that the optimum A/B pattern is obtained within a small number of generations about an order of

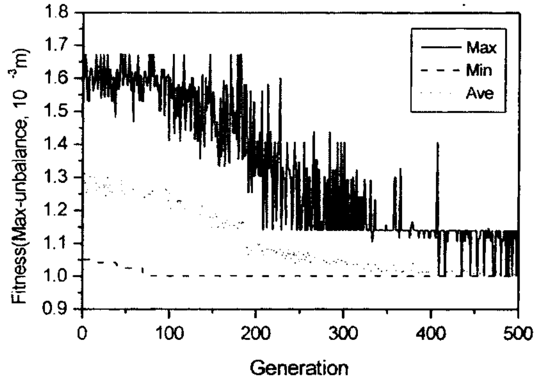


Fig. 2 Convergence plot of the standard genetic algorithm, k_c=8,606 N/m

40. In the ensuing generations, the exponentially decreasing mutations (see Eq. (2)) and constant crossover homogenize the population as demonstrated by the convergence of the mean (and eventually the maximum) value of fitness the population to its minimum value.

Once the "best" intentionally mistuned disk was obtained by GA, its sensitivity to unintentional random mistuning was assessed by adding various levels of random mistuning and evaluating the largest response over the resulting population of disks (10,000 disks). This effort was conducted at the frequency that yielded the largest response of the purely intentionally mistuned disk. In this paper, mistuning is introduced by allowing the blade-to-ground stiffness (k_t) to vary from blade to blade while all other structural parameters remain constant across the disk. Specially, the parameters k_j shown in Fig. 1 are selected to be random variables with mean k_t (= 430,300 N/m) and standard deviation (σ) in unintentional random mistuning. We changed the standard deviation, σ, to vary the unintentional random mistuning level in intentionally mistuned bladed disk.

The value of the coupling stiffness k_c was first set to 45,430 N/m to represent an average to large blade-to-blade coupling level and the two sets of blades A and B were selected to have natural frequencies 5% lower and 5% higher, respectively, than the tuned ones (type C), physically. Mathematically, the stiffness (k_t) of A and B type

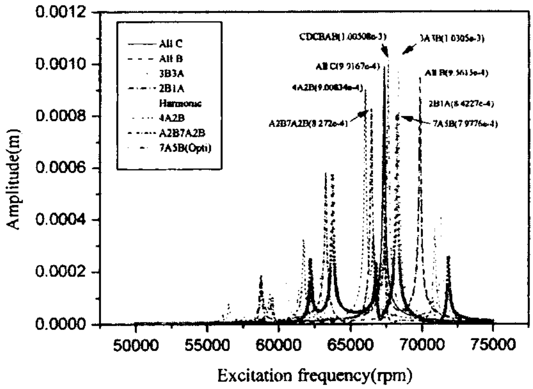


Fig. 3 Comparison with forced response of optimum, tuned and other A/B pattern (k_c=45,430 N/m)

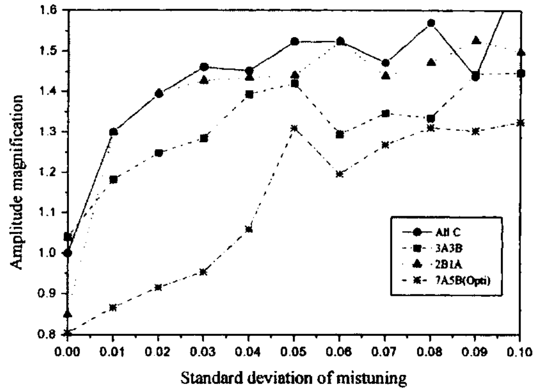


Fig. 4 Amplification factor of the forced response with respect to the tuned disk as a function of the level of unintentional random mistuning (k_c=45,430 N/m)

blade is selected to have 10% lower and 10% higher than the tuned ones. Under these conditions, the genetic optimization algorithm found the optimum configuration of blade as 7A5B. Fig. 3 shows the forced response of optimum, tuned and other A/B patterns of interest here. The notation 3A3B refers to disks formed of 2 groupings AAABBB or $(AAABBB)_2$ and similarly 2B1A represents $(BBA)_4$. The largest amplitude of blade response of this pattern experiences 0.79 times than the tuned one. Therefore, it is identified that 7A5B is better then tuned and harmonic patterns in the forced response. To assess the sensitivity of this and other noted A/B patterns for unintentional random mistuning, the simulation effort described above was undertaken in the range of 0~10% of stiffness mistuning, i.e., 0~5% frequency mistuning. These results, shown in Fig. 4, clearly demonstrate the benefit of using the 7A5B pattern of intentional mistuning, especially at reasonably low levels of unintentional random mistuning. For example, with 3% of unintentional random mistuning, the largest magnification observed on the tuned disk is 1.45 vs. 0.95 for the 7A5B one! At higher mistuning level, the different curves become closer but the one corresponding to the 7A5B disk remains the lowest one.

Other A/B patterns of interest here are the 2B1A and 3A3B whose amplitude magnification curves are also shown in Fig. 4. Interestingly, it is seen that the pattern 2B1A which does lead to a large reduction (18%) of the response in the absence of unintentional random mistuning, namely $\sigma=0$, also exhibits a very large sensitivity to this random mistuning and performs approximately as the tuned disk for $\sigma\neq0$. On the contrary, the pattern 3A3B displays a larger response than the tuned disk for $\sigma=0$ but is much less sensitive to the effects of unintentional random mistuning and thus yields smaller amplitude magnifications for $\sigma=[0.01, 0.05]$.

Also the probability density function for optimum pattern 7A5B, 2B1A and tuned cases is considered as shown in Fig. 5 with 1% unintentional random mistuning. The vertical lines in Fig. 5 indicate the amplitude of each pattern without unintentional random mistuning. For example, in case of 7A5B, the "94.94th percentile" means that 94.94 percent of all randomly mistuned rotors with 1% unintentional random mistuning are expected to exhibit lower amplitude than that without unintentional random mistuning. According to this figure, the probability that each pattern can have larger amplitude than that without unintentional random mistuning is 5.06%, 29.54% and 42.96%, respectively. Therefore, it is also identified that the sensitivity of optimum pattern 7A5B to unintentional random mistuning is lower than other A/B pattern and tuned blade.

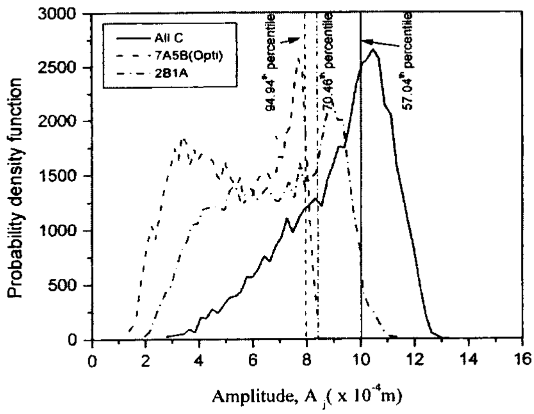


Fig. 5 Probability density function, (k_c=45,430 N/m)

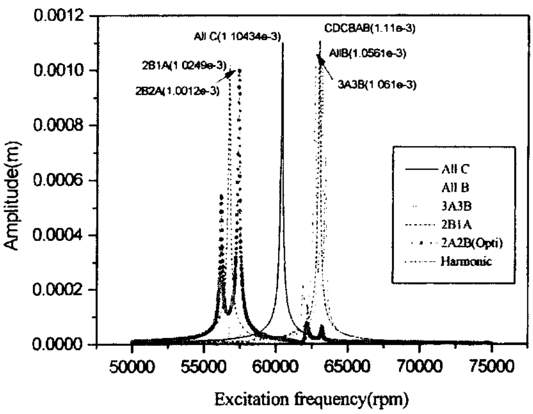


Fig. 6 Comparison with forced response of optimum, tuned and other A/B pattern (k_c=8,606 N/m)

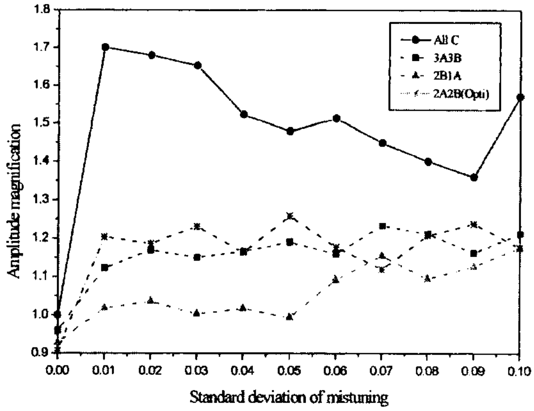


Fig. 7 Amplification factor of the forced response with respect to the tuned disk as a function of the level of unintentional random mistuning (k_c=8,606 N/m)

These effort were repeated for weakly coupled blades, k_c=8,606 N/m and led to the optimum pattern 2A2B. The forced response of optimum, tuned and other A/B patterns of interest here are shown in Fig. 6. Optimum pattern 2A2B have smallest amplitude in the frequency range. As in the previous case, this computation was followed by a sensitivity analysis to unintentional random mistuning. Specifically, the amplitude magnification factor obtained in a 10,000-disk Monte Carlo simulation was plotted as a function of the standard deviation of frequency mistuning for the tuned configuration and various A/B patterns, see Fig. 7. As in Fig. 4, note first, the large reduction in the amplitude magnification obtained by selecting the 2A2B pattern over the tuned one (all C), i.e., from 1.72 to 1.22 at low standard deviation of mistuning. Interestingly, it is seen from Fig. 7 that the optimal solution, 2A2B, does not yield the lowest amplitude magnification when unintentional random mistuning is present. Indeed, the curves corresponding to the systems 3A3B and 2B1A are generally slightly below their 2A2B counterpart for $\sigma \neq 0$. Nevertheless, the amplitude magnification is always less for the optimal A/B pattern than it is for the tuned disk.

In Fig. 7, the standard deviation of unintentional random mistuning at which the maximum magnification occurs tends to be pushed toward higher levels. It occurs at 1%~2% without intentional mistuning (All C) and at 7% with intentional mistuning which creates an additional benefit when the variability of the frequencies of the manufactured blades is low. It means that the response penalty for achieving very tight manufacturing control of the geometric parameters of the airfoils has been substantially reduced.

In Fig. 8, the probability density function for optimum pattern 2B2A, 2B1A and tuned cases is also shown with 1% unintentional random mistuning. According to this figure, the probability that each pattern exhibit larger amplitude than that without unintentional random mistuning is

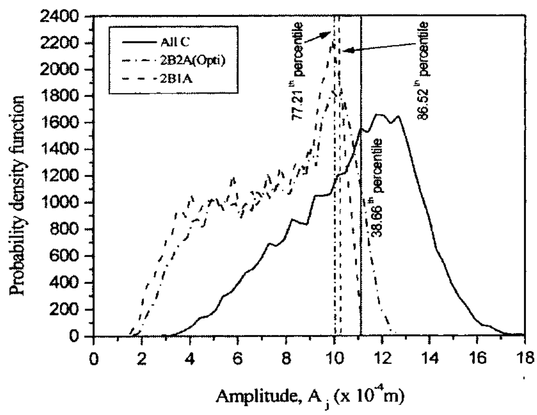


Fig. 8 Probability density function, (k_c=8,606 N/m)

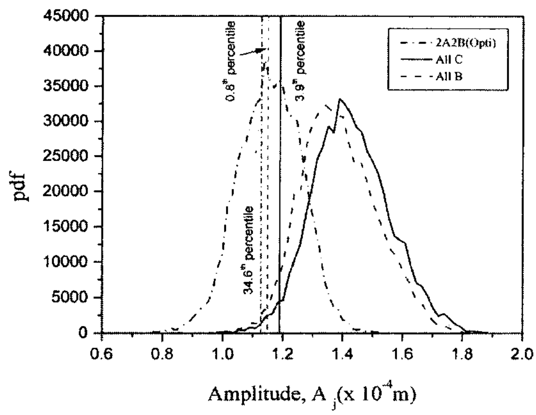


Fig. 10 Probability density function (two-degree-of-freedom model)

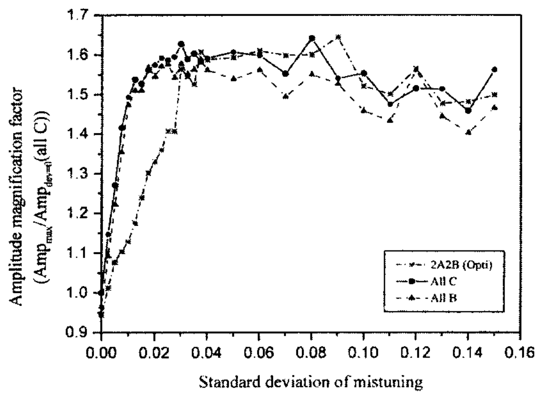


Fig. 9 Amplification factor of the forced response with respect to the tuned disk as a function of the level of unintentional random mistuning (two-degree-of-freedom model)

22.79%, 13.48% and 61.34%, respectively. Therefore, it is also identified that the sensitivity of 2B1A to unintentional random mistuning is lower than optimum solution, 2A2B and tuned blade when unintentional random mistuning is present as in Fig. 7.

Figure 9 shows the amplification of the forced response with respect to the tuned disk as a function of the level of unintentional random mistuning by two-degree-of-freedom model (Mignolet at el., 1999). The details about two-degree-of-freedom model are omitted here because these are well shown in the reference. In this figure, blades C also is tuned, A and B have natural frequencies 4% lower and higher, respectively, than tuned. Under these conditions, the genetic optimization algorithm found the optimum configuration of blade as 2A2B. The largest amplitude of blade response of this pattern experiences 0.95 times than the tuned one. In Fig. 9, with 1% of unintentional random mistuning, the largest magnification observed on the tuned disk is 1.49 vs. 1.12 for the 2A2B. Therefore, optimum pattern 2A2B can have better manufacturing control margin than tuned system until with 3% unintentional random mistuning.

The probability density function for two-degree-of-freedom model is shown in Fig. 10. In tuned cases, the sensitivity to unintentional random mistuning is extremely large because just only 0.8 and 3.9 percent of all randomly mistuned rotors with 2% unintentional random mistuning are expected to exhibit lower amplitude than that without unintentional random mistuning as shown in Fig. 9.

Additional study cases with the model of Fig. 1 and a related two-degree-of-freedom per blade system (Mignolet at el., 1999) confirm the general observation stated above.

These exciting computational results clearly demonstrate the potential usefulness of intentional mistuning for the reduction of the sensitivity of bladed disks to unintentional random mistuning and naturally prompt for a physical explanation of the benefit. Since the base system (without unintentional random mistuning) is not tuned, most of the phenomenological findings available in the literature are not applicable to the present situation and one must resort to a revisit of fundamental concepts and analysis tools. For example, an analogy between bladed disks and infinite chains has often been relied upon (Wei and Pierre, 1988a ; Castanier and Pierre, 1993, 1997 ; LaBorde, 1999) to understand the mechanisms of propagation of energy around the disk. First and foremost in this area is the passband/stopband structure of the chain which conveniently maps the natural frequencies of that infinite system. Specifically, a harmonic disturbance of a frequençy located in a passband will induce a wave that propagates through the entire chain. On the contrary, if the frequency is in the stopband, the effect of this same disturbance will be confined to a small portion of the chain. This discussion has traditionally been limited to the tuned system (Wei and Pierre, 1988a ; Castanier and Pierre, 1993, 1997 ; LaBorde, 1999) but LaBorde(1999) extended these concepts to mistuned bladed disks by "unwrapping" a specific disk and repeating its pattern indefinitely. This process then creates an infinite, periodic chain the natural frequencies of which can be determined by assuming that the response of the component $j+N$ is equal to its jth counterpart but with a phase shift $\phi \in [0, \pi]$. For each value of this phase angle, N natural frequencies are obtained leading to the complex plots of Fig. 11 and 12. On this figure, the passbands are the domains of the vertical axes corresponding to the finite segments while the stopbands correspond to the gaps between segments. Finally, the natural frequencies of the disk are obtained for $\phi=0$.

Figure 11 displays the passband/stopband structure for both a tuned disk (continuous lines) and one exhibiting a small mistuning (dashed lines). Note that the tuned disk exhibits a single broad passband as the various segments connect to each other. Further, mistuning fractions of this domain into N separate passbands and stopbands naturally occur around the natural frequencies of the tuned disk. This observation implies that any disturbance of such frequency will not generate a wave, as for the tuned system, but rather will have a confined effect thereby demonstrating the expected occurrence of localization of the forced response. As for the severity of the localization and the magnification of the amplitude of forced

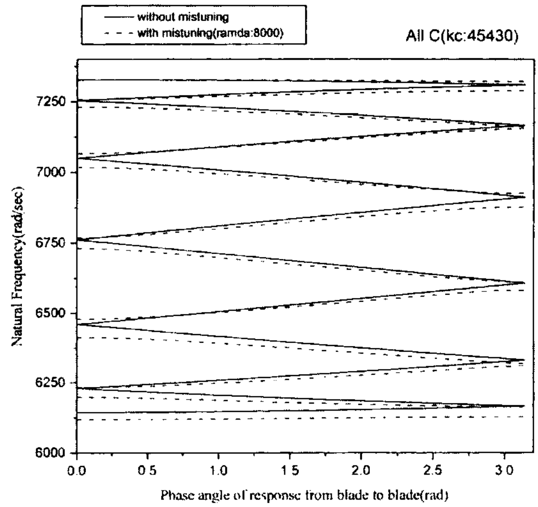


Fig. 11 Passband/stopband structure of the tuned system without (full line) and with (dashed line) unintentional random mistuning, k_c= 45,430 N/m

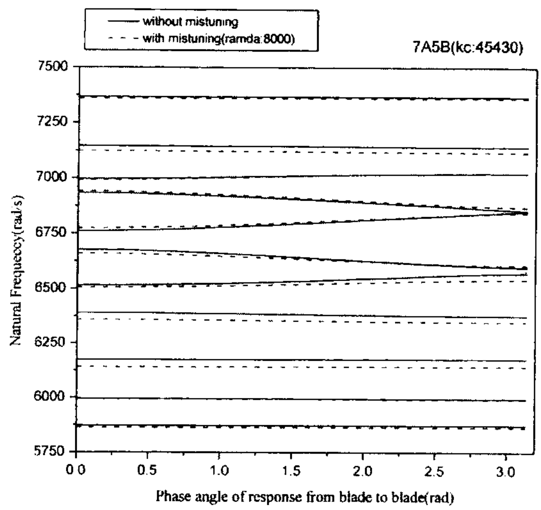


Fig. 12 Passband/stopband structure of the optimally intentional mistuned system 7A5B without (full line) and with (dashed line) unintentional random mistuning, k_c=45,430 N/m

response, it has been stated that the largest amplifications are possible only through an energy exchange between various closely spaced modes.

In light of these comments, note the passband/stopband structure of the optimally intentionally mistuned disk 7A5B shown in Fig. 12. The excitation frequency at which the peak response occurs is 7148 rad/s, which is at the edge of a very narrow passband completely isolated from all other ones. Thus, with an additional unintentional random mistuning, the passband only exhibits a small shift and no other mode is likely to be substantially excited by the 7148 rad/s excitation. From this graph, it is then expected that unintentional random mistuning would not produce any significant effect as demonstrated by the Monte Carlo simulations of Fig. 4.

5. Summary

The investigation of this paper is concerned with the use of intentional mistuning of bladed disks to reduce their sensitivity to an unintentional random mistuning, which can be occurring in the manufacturing process or in service wear. More specifically, it was proposed, for complexity/cost reasons, to limit the number of different blades to two (A and B, say) and to optimize the arrangement of these blades around the disk to reduce the effects of unintentional random mistuning.

The strategy proposed to address this optimization problem relies on the following two steps :

(i) optimization of the arrangement of A and B blades to yield patterns that exhibit small/the smallest maximum amplitudes of forced response of the disk in the absence of unintentional random mistuning by Genetic Algorithm.

(ii) a qualitative/quantitative estimate of the sensitivity for the optimized intentionally mistuned bladed disks with respect to unintentional random mistuning, which was performed by analyzing their amplification factor, probability density function and passband/stopband structures.

In regards to its overall goals, the findings of this investigation can be summarized as follows.

(1) The optimized pattern may or may not appear as variations of simple harmonic patterns : the patterns 7A5B, 3A3B, 2A2B, 2A1B, on the 12-blade disk may appear as distorted 1, 2, 3, and 4 harmonics of mistuning but the distortion plays an important role in reducing the amplitude magnification. It would appear that patterns of blades close to simple harmonics are in general less robust to unintentional random mistuning than those that are not.

(2) The A/B pattern yielding the smallest maximum amplitude of response in the absence of unintentional random mistuning is not always optimal when unintentional random mistuning is factored in but the amplitude magnification of this A/B pattern is always less, most often much less, than that of the tuned system.

(3) Genetic algorithms were shown to be applicable and computationally attractive in the search for intentional mistuning patterns consisting of only two types of blades that produce the largest reduction in the forced response of the disk to a given engine order excitation.

The following specific observations were also made.

(4) The largest amplitude on the disk in the absence of mistuning can be reduced by up to 20～30% by using a disk with an A/B blade pattern as opposed to a tuned one, see Figs. 3～10.

(5) The amplitude magnification due to unintentional random mistuning is often dramatically reduced. For example, in Fig. 7, the largest amplification due to unintentional random mistuning is 1.72 (largest amplitude$=1.72\times$tuned) without intentional mistuning but it is only 1.1 at that same standard deviation of random mistuning with an A/B pattern.

(6) The standard deviation of unintentional random mistuning at which the maximum magnification occurs tends to be pushed toward higher levels. That is, the response penalty for achieving very tight manufacturing control of the geometric parameters of the airfoils has been substantially reduced.

(7) The sensitivity and robustness of the

optimized intentionally mistuned bladed disks with respect to unintentional random mistuning is identified by comparing their amplification factor as a function of the level of unintentional random mistuning, probability density function and passband/stopband structures with other patterns and tuned blade in Figs 4～10.

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